Please email me if you are interested in the classroom materials and additional resources.  Thank you!

NCTM San Diego 2019
Ralph Pantozzi
Kent Place School
Summit NJ
@mathillustrated
mathillustrated@verizon.net

E! THS

TRUE HOLLYWOOD STORY

NCTM San Diego 2019

Ralph Pantozzi

Kent Place School

Summit NJ

@mathillustrated

mathillustrated@verizon.net

e! TPS

TRUE PRECALCULUS STORY

NCTM San Diego 2019
Ralph Pantozzi
Kent Place School
Summit NJ
@mathillustrated
mathillustrated@verizon.net

samjshah @samjshah 1d

Does anyone have any magical motivation for stumbling upon "e" in an algebra 2 class, other than compound interest? #MTBoS

Little did I know I was going to get so many responses! I wanted to archive them here, which is why I'm writing this post, but then share where I'm landing on this whole "e" thing right now.

**Ways To Introduce "e"**

https://samjshah.com/2019/03/05/the-challenge-that-e-poses/

*e*nliven the introduction to *e* by telling its true story. It’s one more *e*nteresting than compound interest. We’ll *e*xamine average rates of change, series, Pascal's triangle, limits, probability, and even *e*uler's method with multiple representations in Precalculus.

Leave your student audience *e*ager to learn more about a number with many claims to fame.

Some numbers have it easy – like that number

that relates the circumference

and the diameter of a circle.

*e*, however, has is own *e*nteresting story.

## How does your presentation align with NCTM's dedication to equity and access?

The activities presented have been designed to invite students to pose questions, promoting students' engagement with math and their belief that their own ideas are at the center of what it means to do mathematics.

By presenting this challenging math content in ways that draw upon students' prior knowledge, mathematical connections, and multiple representations, the lessons provide students with multiple entry points and different possible forms of understanding.

*e*veryone in the room has a number.

I'm going to call out random numbers

from 1 to *n*.

If you number is called, stand up.

What do you *e*magine will happen?

What do you wonder?

This slide intentionally left blank.

**An Expected Value Problem**

Harris S. Shultz, California State University, Fullerton, CA

If numbers are randomly chosen from the interval [0, 1], the expected number of selections necessary until the sum of the chosen numbers first exceeds 1 is $e$ [see Amer. Math. Monthly, 68 (1961) 18–33]. Here we give a purely elementary proof that if $X$ is the smallest $r$ satisfying $t_1 + t_2 + \cdots + t_r > 1$ for randomly selected (with replacement) $t_1, t_2, \ldots$ in the set $\{1/n, 2/n, \ldots, 1\}$, then $E(X) = (1 + (1/n))^n$. Thus, $E(X)$ approximates $e$ for large $n$.

An Expected Value Problem
Author(s): Harris S. Shultz
Source: *The Two-Year College Mathematics Journal*, Vol. 10, No. 4 (Sep., 1979), pp. 277-278
Published by: Taylor & Francis, Ltd. on behalf of the Mathematical Association of America
Stable URL: https://www.jstor.org/stable/3026626
Accessed: 24-03-2019 18:56 UTC

It is more convenient to proceed as follows: Randomly select, with replacement, $t_1, t_2, \ldots$ from the set $\{1, 2, \ldots, n\}$, and let $X$ be the smallest $r$ for which $t_1 + t_2 + \cdots + t_r > n$. Clearly, $t_1 + t_2 + \cdots + t_{n+1} \geqslant n - 1 > n$, so that $X \leqslant n + 1$. Consider $P(X \geqslant j + 1)$ for $1 \leqslant j \leqslant n$. If $X > j$, then $t_1 + t_2 + \cdots + t_j \leqslant n$. Furthermore, $\phi(t_1, t_2, \ldots, t_j) = (t_1, t_1 + t_2, \ldots, t_1 + t_2 + \cdots + t_j)$ is a strictly increasing $j$-tuple with values in $\{1, 2, \ldots, n\}$. Since $\phi$ is a 1-1 mapping from $\{(t_1, t_2, \ldots, t_j) : \sum_{i=1}^{j} t_i \leqslant n\}$ onto the set of strictly increasing $j$-tuples in $\{1, 2, \ldots, n\}$, and since there exist exactly $\binom{n}{j}$ such strictly increasing $j$-tuples in $\{1, 2, \ldots, n\}$, there are exactly $\binom{n}{j}$ $j$-tuples $(t_1, t_2, \ldots, t_j)$ in $\{1, 2, \ldots, n\}$ such that $\sum_{i=1}^{j} t_i \leqslant n$. Since there are $n^j$ $j$-tuples $(t_1, t_2, \ldots, t_j)$ altogether,

$$P(X \geqslant j + 1) = \binom{n}{j}(1/n^j). \qquad (*)$$

This equation also holds for $j = 0$ since $P(X \geqslant 1) = \binom{n}{0}(1/n^0) = 1$. Furthermore, $2 \leqslant X \leqslant n + 1$ implies that $P(X = 0) = P(X = 1) = 0$. Thus,

$$E(X) = \sum_{k=2}^{n+1} kP(X = k) = \sum_{k=2}^{n+1} k[P(X \geqslant k) - P(X \geqslant k + 1)]$$

$$= \sum_{k=2}^{n+1} kP(X \geqslant k) - \sum_{k=3}^{n+2} (k - 1)P(X \geqslant k).$$

Since $P(X \geqslant n+2) = 0$ and $P(X \geqslant 1) = P(X \geqslant 2) = 1$, we proceed to get

$$E(X) = 2P(X \geqslant 2) + \sum_{k=3}^{n+1} [k-(k-1)] P(X \geqslant k)$$

$$= P(X \geqslant 1) + P(X \geqslant 2) + \sum_{k=3}^{n+1} P(X \geqslant k)$$

$$= \sum_{k=1}^{n+1} P(X \geqslant k) = \sum_{j=0}^{n} P(X \geqslant j+1).$$

Substituting (*) into the above expression yields $E(X) = \sum_{j=0}^{n} \binom{n}{j}(1/n^j) = (1+1/n)^n$.

The number $e$ can arise from students' investigations.

Many presentations of $e$ in textbooks take students directly from compound interest to $e$…

…but we can take a different path.

One that involves students in noticing and wondering about the connections between mathematical topics, and in d*e*eply examining what "continuous" really means in the phrase "continuous compounding".

An *e*arly introduction to *e*

in graphical, symbolic, and numerical ways

can help students be able to *e*xplain what is

going on with *e* in calculus later on.

Some numbers have it easy– like that number

that relates the circumference

and the diameter of a circle.

$e$, however, has its own $e$nteresting story.

I would go as far to say that $e$

is a very r$e$latable number.


e: The Story of a Number
ELI MAOR