

1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	3	1	0	0	0
1	4	6	4	1	0	0
1	5	10	10	5	1	0
1	6	15	20	15	6	1
1	7	21	35	35	21	7
1	8	28	56	70	56	28
1	9	36	84	126	126	84
1	10	45	120	210	252	210


$$\binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

apply this to...

$$\left(1 + \frac{1}{n}\right)^n$$

$$1^n + \frac{n}{1} \left(1\right)^{n-1} \left(\frac{1}{n}\right)^1 + \frac{n \cdot (n-1)}{2 \cdot 1} \left(1\right)^{n-2} \left(\frac{1}{n}\right)^2 +$$

$$\frac{n \cdot (n-1) \cdot (n-2)}{3 \cdot 2 \cdot 1} \left(1\right)^{n-3} \left(\frac{1}{n}\right)^3 +$$

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3)}{4 \cdot 3 \cdot 2 \cdot 1} \left(1\right)^{n-4} \left(\frac{1}{n}\right)^4 + \dots$$

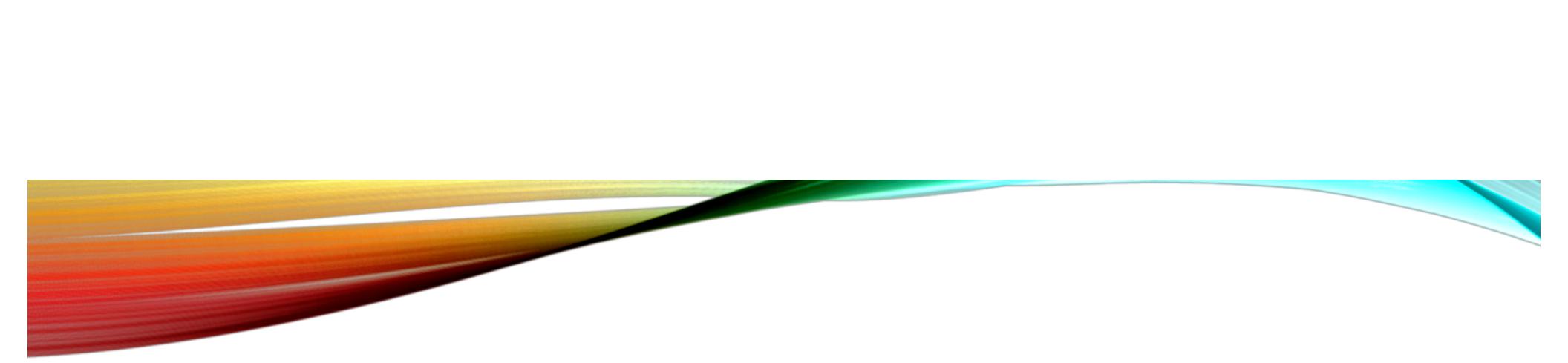

$$1 + 1 + \frac{\left(1 - \frac{1}{n}\right)}{2!} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{3!} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right)}{4!} + \dots$$



As n approaches infinity,

$$1 + 1 + \frac{\left(1 - \frac{1}{n}\right)}{2!} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{3!} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right)}{4!} + \dots$$

approximates



As n approaches infinity,

$$1 + 1 + \frac{\left(1 - \frac{1}{n}\right)}{2!} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{3!} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right)}{4!} + \dots$$

approximates

$$1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

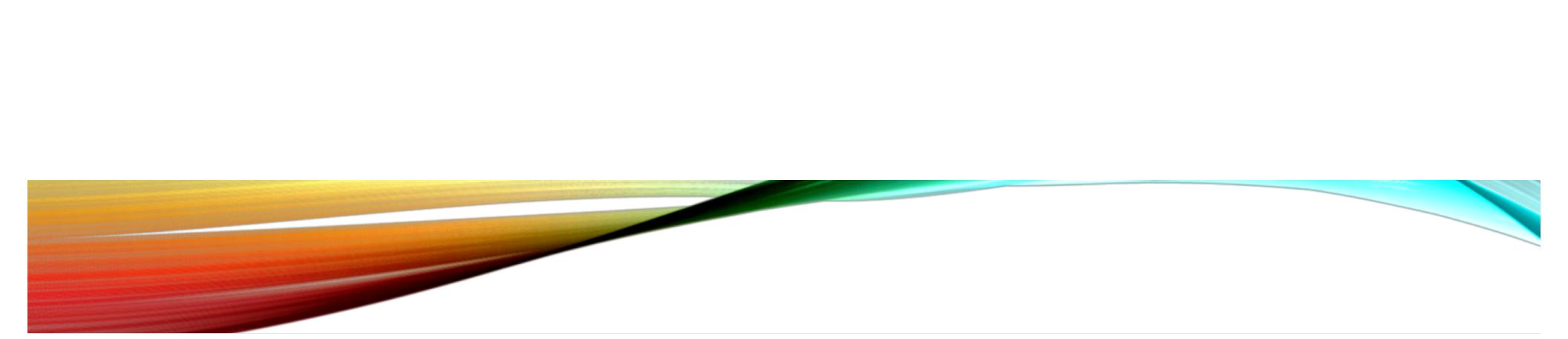

$$\left(1 + \frac{r}{n}\right)^n$$

$$1^n + \frac{n}{1} \left(1\right)^{n-1} \left(\frac{r}{n}\right)^1 + \frac{n \cdot (n-1)}{2 \cdot 1} \left(1\right)^{n-2} \left(\frac{r}{n}\right)^2 + \frac{n \cdot (n-1) \cdot (n-2)}{3 \cdot 2 \cdot 1} \left(1\right)^{n-3} \left(\frac{r}{n}\right)^3 + \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3)}{4 \cdot 3 \cdot 2 \cdot 1} \left(1\right)^{n-4} \left(\frac{r}{n}\right)^4 + \dots$$

$$1 + 1r + \frac{\left(1 - \frac{1}{n}\right)}{2!} r^2 + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{3!} r^3 + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right)}{4!} r^4 + \dots$$

approximates

$$1 + r + \frac{r^2}{2!} + \frac{r^3}{3!} + \frac{r^4}{4!} + \dots$$



How do  $1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots$  and  $1+r+\frac{r}{2!}+\frac{r^2}{3!}+\frac{r^3}{4!}+\dots$  compare?

$1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots$  comes from  $\left(1+\frac{1}{n}\right)^n$

$1+r+\frac{r}{2!}+\frac{r^2}{3!}+\frac{r^3}{4!}+\dots$  comes from  $\left(1+\frac{r}{n}\right)^n$



But remember that  $\left(1 + \frac{1}{n}\right)^r =$

$$1^n + \frac{r}{1} \left(1\right)^{n-1} \left(\frac{1}{n}\right)^1 + \frac{r \cdot (r-1)}{2 \cdot 1} \left(1\right)^{n-2} \left(\frac{1}{n}\right)^2 + \frac{r \cdot (r-1) \cdot (r-2)}{3 \cdot 2 \cdot 1} \left(1\right)^{n-3} \left(\frac{1}{n}\right)^3 + \frac{r \cdot (r-1) \cdot (r-2) \cdot (r-3)}{4 \cdot 3 \cdot 2 \cdot 1} \left(1\right)^{n-4} \left(\frac{1}{n}\right)^4 + \dots$$

which is equivalent to  $1 + \frac{r}{n} + \frac{r \left(\frac{r}{n} - \frac{1}{n}\right)}{2!} + \frac{r \left(\frac{r}{n} - \frac{1}{n}\right) \left(\frac{r}{n} - \frac{2}{n}\right)}{3!} + \frac{r \left(\frac{r}{n} - \frac{1}{n}\right) \left(\frac{r}{n} - \frac{2}{n}\right) \left(\frac{r}{n} - \frac{3}{n}\right)}{4!} + \dots$

which is approximately  $1 + \frac{r}{n}$  for large  $n$



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So,  $\left(\left(1+\frac{1}{n}\right)^r\right)^n$ , or  $\left(1+\frac{1}{n}\right)^{nr}$  is an approximation of  $\left(1+\frac{r}{n}\right)^n$

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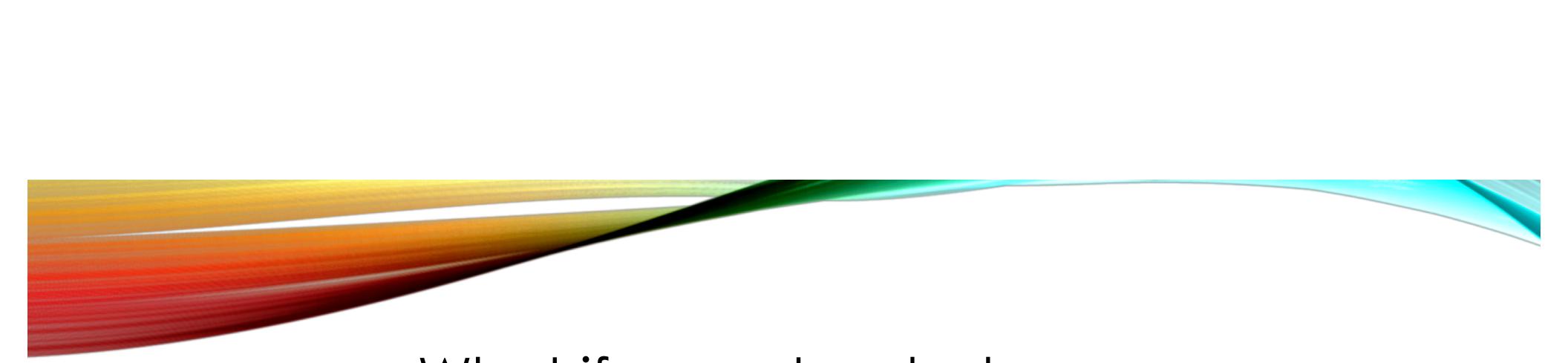
What if we extended  
Pascal's triangle upwards?

1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0
1	2	1	0	0	0	0	0
1	3	3	1	0	0	0	0
1	4	6	4	1	0	0	0
1	5	10	10	5	1	0	0
1	6	15	20	15	6	1	0



What if we extended  
Pascal's triangle upwards?

1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	3	1	0	0	0
1	4	6	4	1	0	0
1	5	10	10	5	1	0
1	6	15	20	15	6	1



What if we extended  
Pascal's triangle upwards?

1	-1	1	-1	1	-1	1
1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	3	1	0	0	0
1	4	6	4	1	0	0
1	5	10	10	5	1	0
1	6	15	20	15	6	1



What if we extended  
Pascal's triangle upwards?

1	-2	3	-4	5	-6	7
1	-1	1	-1	1	-1	1
1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	3	1	0	0	0
1	4	6	4	1	0	0
1	5	10	10	5	1	0
1	6	15	20	15	6	1



1	-3	6	-10	15	-21	28	
1	-2	3	-4	5	-6	7	
1	-1	1	-1	1	-1	1	
1	0	0	0	0	0	0	
1	1	0	0	0	0	0	
1	2	1	0	0	0	0	
1	3	3	1	0	0	0	
1	4	6	4	1	0	0	
1	5	10	10	5	1	0	
1	6	15	20	15	6	1	



1	-4	10	-20	35	-56	84
1	-3	6	-10	15	-21	28
1	-2	3	-4	5	-6	7
1	-1	1	-1	1	-1	1
1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	3	1	0	0	0
1	4	6	4	1	0	0
1	5	10	10	5	1	0
1	6	15	20	15	6	1

1	-4	10	-20	35	-56	84
1	-3	6	-10	15	-21	28
1	-2	3	-4	5	-6	7
1	-1	1	-1	1	-1	1
1	0	0	0	0	0	0
1	1	0	0	0	0	0
1	2	1	0	0	0	0
1	3	3	1	0	0	0

$$\begin{array}{r}
 \overline{1-x+x^2-x^3} \\
 x+1 \overline{) 1} \\
 \underline{-x} \\
 \hline
 1+x \\
 -x \\
 \hline
 -x-x^2 \\
 x^2 \\
 \hline
 x^2+x^3 \\
 -x^3 \\
 \hline
 -x^3-x^4 \\
 x^4
 \end{array}$$



$1 - x + x^2 - x^3 + x^4 - x^5 \dots$  converges to  $\frac{1}{x+1}$  if  $0 < x < 1$ .

Or from the sum of a geometric series:  $\frac{1}{1-r}$   
if  $r = -x$ , then you have  $1 - x + x^2 - x^3 + x^4 - x^5 \dots$



**DELVING**  
deeper

Debra K. Borkovitz

# What's So Special about 3?